FIAN/TD/12/03

Two-column higher spin massless fields in AdS_d

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Abstract

Particular class of AdS_d mixed-symmetry bosonic massless fields corresponding to arbitrary two-column Young tableaux is considered. Unique gauge invariant free actions are found and equations of motion are analyzed.

1 Introduction

In the present paper we consider the problem of manifestly covariant Lagrangian formulation of free mixed-symmetry massless fields propagating on d-dimensional anti-de Sitter background. In contrast to flat spacetime, where free higher spin dynamics is presently well understood [1, 2, 3, 4, 5, 6, 7], the analysis of field dynamics on the AdS_d background is more complicated and up to now only particular examples of mixed-symmetry fields were explicitly analyzed [8, 9, 10, 11, 12, 13]. The reason is that the classification of massless fields in AdS_d is essentially different from that for massless fields in Minkowski space [8]. From the field-theoretical perspective this implies drastic changes for the entire fabric of gauge symmetries as compared to the field dynamics on the flat background [14, 8].

The line of consideration of the present paper is motivated by the approach recently proposed in Ref. [13], which represents mixed-symmetry fields as gauge p-form fields taking values in an appropriate finite-dimensional irreps of AdS_d algebra. The method was illustrated by various examples of mixed-symmetry fields with at most two rows. The goal of the present paper is to apply the general prescription of [13] to higher spin massless fields corresponding to arbitrary two-column Young tableaux.

2 Preliminaries

Consider[1] a Lorentz tensor field $\phi^{(s,p)}(x)$ on d-dimensional spacetime (Minkowski or AdS_d)

$$\phi^{a[s],\,b[p]}(x) \equiv \phi^{[a_1\ldots a_s],\,[b_1\ldots b_p]}(x)\;, \tag{1}$$

which is antisymmetric in both groups of indices, satisfies the Young symmetry condition $\phi^{[\,a_1\ldots a_s,\,a_{s+1}\,]b_2\ldots b_p}(x)=0$, and contains all its traces. The tensor $\phi^{(s,p)}(x)$ will be referred to as the *metric-type* field [13].

The heights of columns s and p are assumed to satisfy the following inequality

$$0<p\leq s\leq \nu\;. \tag{2}$$

A value of the upper bound in (2) is different for massless fields in Minkowski and AdS_d. For the Minkowski space $\nu\equiv\nu_{Mink}=\left[\frac{d-2}{2}\right]$ is a rank of the little Wigner group $SO(d-2)$. For the AdS_d space $\nu\equiv\nu_{AdS}=\left[\frac{d-1}{2}\right]$ is a rank of the vacuum group $SO(d-1)$. It follows that $\nu_{Mink}\leq\nu_{AdS}$, which allows one to conclude that theories of massless fields in AdS_d may give dual descriptions of flat massless fields in the flat limit[2]. If $s>\nu_{AdS}$ and $0<p\leq s$, the corresponding AdS_d higher spin theory is a dual formulation of some particular AdS_d theory with parameters $\tilde{p}$, $\tilde{s}$ from (2).

The metric-type field $\phi^{(s,p)}(x)$ (1) is a gauge field with the gauge transformation law given by two types of gauge parameters [3] (schematically):

$$\delta\phi^{(s,p)}=\partial S^{(s-1,p)}+\partial\Lambda^{(s,p-1)}\;, \tag{3}$$

where tensors $S^{(s-1,p)}$ and $\Lambda^{(s,p-1)}$ are described by Young tableaux obtained by cutting off a cell from the first and the second column of $\phi^{(s,p)}(x)$, respectively, and contain all their traces.

In the sequel, we develop the gauge invariant AdS_d theory for a metric-type field $\phi^{(s,p)}(x)$ with arbitrary values of s and p which satisfy (2). Before plunging into description of our approach, discuss how one may proceed to obtain an AdS_d theory starting form some flat Lagrangian $\mathcal{L}$ invariant under (3). A general prescription consists in replacing $\partial\rightarrow\mathcal{D}$, where $\mathcal{D}$ is the background Lorentz derivative commuting as $[\mathcal{D},\mathcal{D}]\sim\lambda^2$, and adding appropriate mass-like terms in $\mathcal{L}$ which support the gauge invariance. However, as pointed out in [14, 8], it is not always consistent procedure for a generic mixed-symmetry field: only a part of gauge symmetries can be deformed to AdS_d to obtain a unitary theory. In the case under consideration, the symmetry which survives in AdS_d corresponds to the gauge parameter $\Lambda^{(s,p-1)}$. Lack of one of gauge symmetries on AdS_d results in a discrepancy between degrees

[1] Throughout the paper we work within the mostly minus signature and use notations $\underline{m},\underline{n}=0\div d-1$ for world indices, $a,b=0\div d-1$ for tangent $so(d-1,1)$ vector indices and $A,B=0\div d$ for tangent $so(d-1,2)$ vector indices. We also use condensed notations for a set of antisymmetric indices: $a[k]\equiv[a_1\ldots a_k]$. Indices denoted by the same letter are assumed to be antisymmetrized as $X^aY^a\equiv\frac{1}{2!}(X^{a_1}Y^{a_2}-X^{a_2}Y^{a_1})$.

[2] For discussion of dual formulations of free higher spin fields in Minkowski space see papers [15, 16, 6, 17].

of freedom of a field $\phi^{(s,p)}(x)$ on the flat and AdS_d backgrounds. The balance may be restored by introducing a Stueckelberg field for the missing symmetry $S^{(s-1,p)}$ which can be gauged away for $\lambda \neq 0$, however. The flat limit of such extended theory describes not one but two independent fields [8, 11].

Within our approach we do not search for the AdS_d deformation of some Lagrangian describing flat field dynamics. Instead, we start with dynamics on the AdS_d background and investigate its flat limit then.

The background Minkowski or AdS_d geometry is described by the frame field $h^a = h_{\underline{n}}{}^a\, dx^{\underline{n}}$ and Lorentz spin connection $\omega^{ab} = \omega_{\underline{n}}{}^{ab}\, dx^{\underline{n}}$ which obey the equation

$$[\mathcal{D}_{\underline{m}}, \mathcal{D}_{\underline{n}}]\phi^{a[s],\, b[p]} = \lambda^2 (s\ h_{\underline{m}}{}^a\, h_{\underline{n}\, c}\, \phi^{ca[s-1],\, b[p]} + p\ h_{\underline{m}}{}^b\, h_{\underline{n}\, c}\, \phi^{a[s],\, cb[p-1]}) - (\underline{m} \leftrightarrow \underline{n})\ , \quad (4)$$

where

$$\mathcal{D}_{\underline{n}}\phi^{a[s],\, b[p]} = \partial_{\underline{n}}\phi^{a[s],\, b[p]} + s\ \omega_{\underline{n}}{}^a{}_c\, \phi^{ca[s-1],\, b[p]} + p\ \omega_{\underline{n}}{}^b{}_c\, \phi^{a[s],\, cb[p-1]}\ , \quad \partial_{\underline{n}} = \frac{\partial}{\partial x^{\underline{n}}}\ . \quad (5)$$

Also, the zero-torsion condition $\mathcal{D}_{\underline{n}} h_{\underline{m}}{}^a - \mathcal{D}_{\underline{m}} h_{\underline{n}}{}^a = 0$ is imposed. It expresses the spin connection $\omega_{\underline{m}}{}^{ab}$ in terms of the first derivatives of the frame field $h_{\underline{m}}{}^a$. The equation (4) describes AdS_d spacetime with the symmetry algebra $o(d-1,2)$ when $\lambda^2 > 0$. Minkowski space-time corresponds to $\lambda = 0$.

The covariant D'Alembertian is

$$\mathcal{D}^2 \equiv \mathcal{D}^a \mathcal{D}_a = h^a_{\underline{m}} \mathcal{D}^{\underline{m}} (h_{\underline{n},\, a} \mathcal{D}^{\underline{n}})\ , \quad (6)$$

where the background Lorentz covariant derivative is given by (5).

In what follows we use AdS_d covariant notations and operate with AdS_d tensors $T^{A[m]}(x)$. To relate Lorentz and AdS_d covariant realizations we introduce a compensator vector $V^A(x)$ normalized as $V^A V_A = 1$ [18]. This allows one to identify the Lorentz subalgebra $so(d-1,1)$ within the AdS algebra $so(d-1,2)$ as the stability algebra of the compensator, which results in the covariant splitting of the $so(d-1,2)$ 1-form connection $\Omega^{[AB]}$ into the frame field E^A and the Lorentz connection $\omega^{[AB]}$: $E^A = DV^A \equiv dV^A + \Omega^{AB} V_B$, $\omega^{[AB]} = \Omega^{[AB]} - 2\lambda\, E^{[A}\, V^{B]}$ [18]. In these notations, the background AdS_d geometry (h^A, $\omega_0^{[AB]}$) is defined by the "zero-curvature" condition [19]

$$R^{AB}(\Omega_0) \equiv d\Omega_0^{AB} + \Omega_0{}^A{}_C \wedge \Omega_0{}^{CB} = 0\ . \quad (7)$$

The action of the background covariant derivative on an arbitrary AdS_d tensor is given by

$$D_0 T^{A[m]} = dT^{A[m]} + m\, \Omega^A_{0\,C} \wedge T^{CA[m-1]}\ . \quad (8)$$

3 p-form gauge fields

According to the general prescription of Ref. [13], we describe two-column mixed-symmetry field $\phi^{(s,p)}$ (1) propagating on the AdS_d space in terms of a *frame-type* p-form gauge field

$$e^{a[s]}_{(p)} = e^{[a_1 \ldots a_s];\, [\underline{m}_1 \ldots \underline{m}_p]}\ dx_{\underline{m}_1} \wedge \ldots \wedge dx_{\underline{m}_p}\ . \quad (9)$$

It is convenient to replace world indices in (9) with tangent indices

$$e^{[a_1\dots a_s];\,[b_1\dots b_p]} \equiv e^{[a_1\dots a_s];\,[\underline{m}_1\dots\underline{m}_p]}\; h_{\underline{m}_1}{}^{b_1}\dots h_{\underline{m}_p}{}^{b_p}\;, \tag{10}$$

where $h_{\underline{m}}{}^{a}$ is the background frame field in AdS_d.

The frame-type p-form gauge field (9) gives rise to a collection of components arising through tensoring of world and tangent indices

$$e^{a[s];\,b[p]} \sim \bigoplus_{i=0}^{p} \phi^{a[s+i],\,b[p-i]}\;. \tag{11}$$

Here each tensor component $\phi^{a[s+i],\,b[p-i]}$ corresponds to a Young tableau with $(s+i)$ antisymmetric indices in the first column and $(p-i)$ antisymmetric indices in the second column and contain all its traces. It is convenient to denote each component in (11) as $\phi^{(i)}$. The first component $i=0$ in (11) is identified with the metric-type field $\phi^{(0)}\equiv\phi^{(s,p)}$ (1).

In principle, the p-form field (9) is sufficient to construct a gauge invariant action functional. However, to control gauge symmetries in a manifest manner one should introduce additional p-form gauge fields. In the case under consideration, appropriate set of fields is given by $e^{a[s]}_{(p)}$ and $\omega^{a[s+1]}_{(p)}$, which will be referred to as the physical and the auxiliary p-forms [13].

The Abelian curvature $(p+1)$-forms associated with the physical and the auxiliary p-form gauge fields read as

$$r^{a[s]}_{(p+1)} = \mathcal{D}e^{a[s]}_{(p)} + h_b\wedge\omega^{a[s]b}_{(p)}\;, \qquad \mathcal{R}^{a[s+1]}_{(p+1)} = \mathcal{D}\omega^{a[s+1]}_{(p)} - (s+1)\lambda^2 h^a\wedge e^{a[s]}_{(p)}\;. \tag{12}$$

They are invariant under the gauge transformations with $(p-1)$-form gauge parameters $\Lambda^{a[s]}_{(p-1)}$ and $\xi^{a[s+1]}_{(p-1)}$

$$\delta e^{a[s]}_{(p)} = \mathcal{D}\Lambda^{a[s]}_{(p-1)} + h_b\wedge\xi^{a[s]b}_{(p-1)}\;, \qquad \delta\omega^{a[s+1]}_{(p)} = \mathcal{D}\xi^{a[s+1]}_{(p-1)} - (s+1)\lambda^2 h^a\wedge\Lambda^{a[s]}_{(p-1)}\;. \tag{13}$$

In particular, the structure of the gauge invariance (13) requires the curvatures to satisfy Bianchi identities

$$\mathcal{D}r^{a[s]}_{(p+1)} + h_b\wedge\mathcal{R}^{a[s]b}_{(p+1)} = 0\;, \qquad \mathcal{D}\mathcal{R}^{a[s+1]}_{(p+1)} - (s+1)\lambda^2 h^a\wedge r^{a[s]}_{(p+1)} = 0\;. \tag{14}$$

The gauge transformations (13) are reducible. There is a set of $(l+2)$-th level $(0\le l\le p-2)$ gauge transformations of the form

$$\begin{aligned} \delta\Lambda^{a[s]}_{(p-l-1)} &= \mathcal{D}\Lambda^{a[s]}_{(p-l-2)} + h_b\wedge\xi^{a[s]b}_{(p-l-2)}\;,\\ \delta\xi^{a[s+1]}_{(p-l-1)} &= \mathcal{D}\xi^{a[s+1]}_{(p-l-2)} - (s+1)\lambda^2 h^a\wedge\Lambda^{a[s]}_{(p-l-2)}\;. \end{aligned} \tag{15}$$

The role of the shift parameter $\xi^{a[s+1]}_{(p-1)}$ in the gauge transformations (13) is to compensate all components of the physical p-form field in (11) with $i>0$. It can be easily seen from the decomposition of the gauge parameters $\xi^{a[s+1]}_{(p-1)}$ analogous to (11)

$$\xi^{a[s+1];\,b[p-1]} \sim \bigoplus_{i=0}^{p-1} \xi^{a[s+i+1],\,b[p-i-1]}\;, \tag{16}$$

where tensors in r.h.s. have Young symmetry properties and contain all their traces. Thus, by gauge fixing with the help of the shift parameters $\xi^{a[s+1]}_{(p-1)}$, the p-form gauge field (9) reduces to the component $\phi^{(0)}$ corresponding to $i=0$ in (11) to be identified with the physical metric-type field (1). The derivative part of (13) can be analyzed along the same lines. Namely, introduce the decomposition of the derivative gauge parameters $\Lambda^{a[s]}_{(p-1)}$ analogous to (16)

$$\Lambda^{a[s];\,b[p-1]} \sim \bigoplus_{i=0}^{p-1} \Lambda^{a[s+i],\,b[p-i-1]} \;, \tag{17}$$

where tensors in r.h.s. have Young symmetry properties and contain all their traces. Then, one finds from (11) and (17) that the metric-type field $\phi^{(0)} \equiv \phi^{(s,p)}$ transforms as (schematically)

$$\delta\phi^{(0)} = \mathcal{D}\Lambda^{(0)} \;, \tag{18}$$

where the gauge parameter $\Lambda^{(0)}$ is the first component in (17) with $i=0$ and has the same symmetry type as $\Lambda^{(s,p-1)}$ in (3). As a consequence of the (15), the gauge transformation (18) is reducible up to a p-th level.

It is worth to comment that our approach is formulated in the way it incorporates the gauge symmetry with parameter $\Lambda^{(s,p-1)}$ (3) only, which is the correct gauge symmetry on the AdS_d background. Another type of gauge symmetry with parameter $S^{(s,p-1)}$ (3) is not placed in the AdS_d formulation and appears in the flat limit $\lambda=0$. We shall comment on this phenomenon later.

The Lorentz p-form gauge fields introduced to describe a mixed-symmetry field in AdS_d can be viewed as a result of the decomposition with respect to the Lorentz group of a p-form gauge field carrying an appropriate irreducible representation of $o(d-1,2)$ [13]. In our case, the Lorentz fields $e^{a[s]}_{(p)}$ and $\omega^{a[s+1]}_{(p)}$ result form the following AdS_d p-form field

$$\Omega^{A[s+1]}_{(p)} \sim e^{a[s]}_{(p)} \oplus \omega^{a[s+1]}_{(p)} \;. \tag{19}$$

With the help of the compensator vector discussed in section 2, the isomorphism (19) takes the precise form

$$\Omega^{A[s+1]}_{(p)} = \omega^{A[s+1]}_{(p)} + \lambda\,(s+1)\,V^A\,e^{A[s]}_{(p)} \;, \tag{20}$$

supplemented with the transversality conditions

$$e^{A[s-1]C}_{(p)} V_C = 0 \;, \quad \omega^{A[s]C}_{(p)} V_C = 0 \;. \tag{21}$$

The appearance of the cosmological parameter λ in (20) is motivated by different mass dimensions of the physical and the auxiliary p-forms since on the level of equations of motion the auxiliary field is expressed in terms of the first derivatives of the physical field.

In the AdS_d formalism, the gauge transformation and the curvature are given by $(0 \leq l \leq p-2)$

$$R^{A[s+1]}_{(p+1)} = D_0\Omega^{A[s+1]}_{(p)} \;, \qquad \delta\Omega^{A[s+1]}_{(p)} = D_0\xi^{A[s+1]}_{(p-1)} \;, \quad \delta\xi^{A[s+1]}_{(p-l-1)} = D_0\xi^{A[s+1]}_{(p-l-2)} \;, \tag{22}$$

where

$$R^{A[s+1]}_{(p+1)} \sim r^{a[s]}_{(p+1)} \oplus \mathcal{R}^{a[s+1]}_{(p+1)} \,, \qquad \xi^{A[s+1]}_{(p-l-1)} \sim \Lambda^{a[s]}_{(p-l-1)} \oplus \xi^{a[s+1]}_{(p-l-1)} \tag{23}$$

and the covariant derivative D_0 is evaluated with respect to the background AdS_d connection Ω_0 (8). Bianchi identities

$$D_0 R^{A[s+1]}_{(p+1)} = 0 \tag{24}$$

are the consequence of the zero-curvature condition $D_0^2 = 0$ (7).

It is interesting to note that in $d = 7$ *mod* 4 dimensions one more irreducibility condition on the tangent indices may be imposed. A duality condition may arise if the tangent $so(d-1,2)$ Young tableau is a column of maximal height $s+1 = \frac{d-1}{2}+1$, which may be dualized using Levi-Civita symbol

$$\Omega_{(p)\,A[s+1]} = \pm\frac{1}{(s+1)!}\, \epsilon_{A[s+1]B[s+1]}\, \Omega^{B[s+1]}_{(p)} \,. \tag{25}$$

This condition decomposes $\Omega^{A[s+1]}_{(p)}$ into selfdual and antiselfdual parts and results in the following relation for Lorentz p-form fields

$$\omega_{(p)}{}^{a[s+1]} = \pm\frac{\lambda}{s!}\, \epsilon^{a[s+1]b[s]}\, e_{(p),\, b[s]} \,. \tag{26}$$

This off-shell constraint can be used to express algebraically the auxiliary field in terms of the physical field or *vice versa*. In this case equations of motion will acquire first order form. Note that in $d = 5$ *mod* 4 dimensions the duality condition may be also imposed but then the field $\Omega^{A[s+1]}_{(p)}$ is necessarily complex. The analogous phenomenon was discussed for massive antisymmetric tensor fields on the flat background in [20] and called odd-dimensional selfduality. In the AdS_d theory the role of mass is played by the cosmological constant λ.

4 Dynamics

In what follows we build higher spin action which correctly describes free dynamics of metric-type field $\phi^{(s,p)}$ on the AdS_d background. It was suggested in [13] to look for free action in the MacDowell-Mansouri form [21, 22]. In our case the most general action reads

$$\begin{aligned}\mathcal{S}_2 = \frac{\kappa_1}{\lambda^2}\int_{\mathcal{M}_d} H_{A[2p+2]} \wedge\, R^{A[p+1]B[s-p]}_{(p+1)} \wedge\, R^{A[p+1]}_{(p+1)}{}_{B[s-p]} \\ +\frac{\kappa_2}{\lambda^2}\int_{\mathcal{M}_d} H_{A[2p+2]} \wedge\, R^{A[p+1]B[s-p-1]C}_{(p+1)} \wedge\, R^{A[p+1]}_{(p+1)}{}_{B[s-p-1]}{}^{D} V_C V_D \,,\end{aligned} \tag{27}$$

where $\kappa_{1,2}$ are arbitrary dimensionless constants and the notation is introduced:

$$H_{A[m]} = \epsilon_{A_1\ldots A_m B_{m+1}\ldots B_{d+1}} h^{B_{m+1}} \wedge \ldots \wedge h^{B_d} V^{B_{d+1}} \,. \tag{28}$$

The freedom in $\kappa_{1,2}$ can be fixed up to an overall factor in front of the action (27) by adding a unique total derivative term

$$\mathcal{O} = \int_{\mathcal{M}_d} d\Big(H_{A[2p+3]} \wedge R_{(p+1)}^{A[p+2]B[s-p-1]} \wedge R_{(p+1)}^{A[p+1]}{}_{B[s-p-1]}{}^{C} V_C \Big) \,. \qquad (29)$$

Note that the form of the action imposes a natural restriction $p \leq [\frac{d-2}{2}]$, while s is not restricted at all. It is tempting to speculate that for $s > \nu_{AdS}$, the action (27) gives rise to dual formulations of two-column metric-type fields in AdS_d.

The variation of the action (27) gives rise to the equations of motion

$$\epsilon^{A[p+1]}{}_{B[p+1]C[d-2p-1]} h^{C_1} \wedge \ldots \wedge h^{C_{d-2p-1}} \wedge R_{(p+1)}^{A[s-p]B[p+1]} = 0 \,. \qquad (30)$$

To clarify the dynamical content hidden in these equations it is convenient to perform analysis in terms of Lorentz components. To this end, introduce the decomposition of the curvature tensors (12) analogous to (11)

$$r^{a[s];\, b[p+1]} \sim \bigoplus_{i=0}^{p+1} r^{a[s+i],\, b[p-i+1]} \,, \qquad (31)$$

$$\mathcal{R}^{a[s+1];\, b[p+1]} \sim \bigoplus_{i=0}^{p+1} \mathcal{R}^{a[s+i+1],\, b[p-i+1]} \,, \qquad (32)$$

where tensors in r.h.s.-s have Young symmetry properties and contain all their traces. By direct calculation one shows that equations (30) can be cast into the form

$$\begin{aligned} p < s: &\quad r^{a[s]}_{(p+1)} = h_{b_1} \wedge \ldots \wedge h_{b_{p+1}}\, T^{a[s],\, b[p+1]} \,, \\ p = s: &\quad r^{a[s]}_{(p+1)} = 0 \end{aligned} \qquad (33)$$

and

$$p \leq s: \quad \mathcal{R}^{a[s+1]}_{(p+1)} = h_{b_1} \wedge \ldots \wedge h_{b_{p+1}}\, C^{a[s+1],\, b[p+1]} \qquad (34)$$

with arbitrary traceless 0-forms $T^{a[s],\, b[p+1]}$ and $C^{a[s+1],\, b[p+1]}$ described by two-column Young tableaux. All other components of the curvatures (31)-(32) are zero. One should note that the tracelessness for $T^{a[s],\, b[p+1]}$ is required only when $\lambda \neq 0$ (it follows from Bianchi identities):

$$\lambda\, T^{a[s-1]c,\, b[p]d} \eta_{cd} = 0 \,. \qquad (35)$$

We shall discuss the distinguished role of this condition later.

In the case $p = s$, which corresponds to the physical metric-type field described by rectangular Young tableau, 0-form C is the *primary* Weyl tensor [13]. It parameterizes those components of the curvatures which are non-vanishing on the mass-shell and cannot be expressed through derivatives of some other curvature components. In the case of the physical metric-type field described by non-rectangular Young tableau with $p < s$, the *primary* Weyl tensor is given by 0-form T, while the 0-form C is the *secondary* Weyl tensor expressed through the first derivatives of the

primary Weyl tensor T by virtue of Bianchi identities [13]. The structure of Weyl tensors, both primary and secondary, is in agreement with Young symmetry types of invariant curvature tensors found in [7] in the framework of non-local formulation of Minkowski higher spin dynamics.

Turn now to the explicit analysis of the equations (33)-(34). By analogy with the decomposition (11), introduce the decomposition of the auxiliary p-form:

$$\omega^{a[s+1];\,b[p]} \sim \bigoplus_{i=0}^{p} \omega^{a[s+i+1],\,b[p-i]} \;. \tag{36}$$

Then, one shows that all components $\omega^{(i)}$ (36) of the auxiliary p-form can be expressed in terms of the first derivatives of the components $\phi^{(i)}$, $0 \leq i \leq p$ of the physical p-from (schematically):

$$\omega^{(i)} = \mathcal{D}\phi^{(i)} + \mathcal{D}\phi^{(i+1)} \;, \qquad 0 \leq i \leq p \;. \tag{37}$$

This expression follows from the fact that the auxiliary field enters the physical curvature (12) without derivatives and the equations (33) are, in fact, linear homogeneous equations with respect to the components of the auxiliary p-form. Notice that when $p < s$, the curvature component $T^{a[s],\,b[p+1]}$ is not zero and does not contain any components of the auxiliary p-form (36). Therefore, the number of linear homogeneous equations (33) matches exactly the number of components of the auxiliary p-form. In other words, Eq.(33) is the constraint, which allows one to express the auxiliary field in terms of the first derivatives of the physical field.

As discussed in section 3, the specific feature of the gauge parameter $\xi^{a[s+1]}_{(p-1)}$ is that it enters the gauge transformations (13) algebraically for the dynamical p-form and through a derivative for the auxiliary p-form. From (16) it follows that the gauge transformations (13) can be cast into the schematic form

$$\begin{aligned} &\delta\phi^{(i)} = \xi^{(i-1)} \;, \qquad 1 \leq i \leq p \;, \\ &\delta\omega^{(j)} = \mathcal{D}\xi^{(j)} + \mathcal{D}\xi^{(j-1)} \;, \qquad 0 \leq j \leq p \;. \end{aligned} \tag{38}$$

Combining these expressions with (37) one finds that, by gauge fixing all redundant components of the physical p-form field to zero, the auxiliary field expresses through the physical one as

$$\omega^{(0)} = \mathcal{D}\phi^{(0)} \;. \tag{39}$$

As a consequence of (34), the second-order dynamical equations of motion on the physical metric-type component $\phi^{(s,p)}$ emerge as the trace of the component $\mathcal{R}^{(0)}$ (32)

$$\mathcal{R}^{a[s]c\,,}{}_{c}{}^{b[p]} = 0 \;, \tag{40}$$

where the auxiliary field is expressed through the first derivatives of the physical one according to (39). Clearly, the tensor in l.h.s. of (40) has the same Young symmetry type as that of the metric-type field $\phi^{(s,p)}$.

To find the explicit form of dynamical equations of motion, we simplify our calculations introducing Fock space notations for metric-type fields

$$|\phi\rangle = \phi_{[a_1\dots a_s],\,[b_1\dots b_p]}\;\alpha_1^{a_1}\dots\alpha_1^{a_s}\alpha_2^{b_1}\dots\alpha_2^{b_p}|0\rangle\;, \tag{41}$$

where α_i^a and $\bar\alpha_j^b,\;\; i,j=1,2$ are creation and annihilation operators defined on the Fock vacuum

$$\bar\alpha_c^i|0\rangle = 0 \tag{42}$$

and satisfying the algebra

$$\{\bar\alpha_a^i,\alpha_b^j\} = \delta^{ij}\eta_{ab}\;,\quad \{\alpha_a^i,\alpha_b^j\} = 0\;,\quad \{\bar\alpha_a^i,\bar\alpha_b^j\} = 0\;. \tag{43}$$

The following notations are convenient in practice

$$\begin{gathered} L_{ij} = \bar\alpha_i^a\bar\alpha_{j\,a}\;,\quad T_{ij} = \alpha_i^a\bar\alpha_{j\,a}\;,\qquad N_{ij} = \alpha_i^a\alpha_{j\,a}\;,\\ D_i = \alpha_i^a\,\mathcal{D}_a\;,\quad \bar D_i = \bar\alpha_i^a\,\mathcal{D}_a\;. \end{gathered} \tag{44}$$

Vector $|\phi\rangle$ satisfies the conditions

$$T_{12}|\phi\rangle = 0\;,\quad T_{11}|\phi\rangle = s_1|\phi\rangle\;,\quad T_{22}|\phi\rangle = s_2|\phi\rangle\;, \tag{45}$$

which reflect its Young symmetry properties.

The physical metric-type field (1) expressed through the p-form field (9) $|e\rangle = e_{[a_1\dots a_s];\,[b_1\dots b_p]}\;\alpha_1^{a_1}\dots\alpha_1^{a_s}\alpha_2^{b_1}\dots\alpha_2^{b_p}|0\rangle$ has the form

$$|\phi\rangle = \sum_{k=0}^{p}(-)^k\frac{(s-k)!}{k!}\,\alpha_1^{a_1}\dots\alpha_1^{a_k}\,\alpha_2^{b_1}\dots\alpha_2^{b_k}\,\bar\alpha_{1\,b_1}\dots\bar\alpha_{1\,b_k}\,\bar\alpha_{2\,a_1}\dots\bar\alpha_{2\,a_k}\,|e\rangle\;. \tag{46}$$

With the help of operators (44) the auxiliary field (39) and the gauge transformation (18) acquire exact form

$$|\omega\rangle = D_1|\phi\rangle\;, \tag{47}$$

$$\delta|\phi\rangle = \Big(D_2 - \frac{1}{s-p+1}\,D_1T_{21}\Big)|\Lambda\rangle\;. \tag{48}$$

The $i=0$ component $|\mathcal{R}^{(0)}\rangle$ of the decomposition (32) is

$$|\mathcal{R}^{(0)}\rangle = \Big(D_2 - \frac{1}{s-p+1}\,D_1T_{21}\Big)|\omega\rangle - \lambda^2(s-p+2)N_{12}|\phi\rangle\;. \tag{49}$$

Substituting the expression for the auxiliary field (47) into the curvature (49) and taking trace $L_{12}|\mathcal{R}^{(0)}\rangle = 0$ (40) we find the equation of motion

$$\Big(\mathcal{D}^2 - D_1\bar D_1 - D_2\bar D_2 - D_1D_2L_{12} + \lambda^2N_{12}L_{12} + \lambda^2(d(p-1)+2p-p^2+s)\Big)|\phi\rangle = 0\;. \tag{50}$$

The covariant D'Alembertian $\mathcal{D}^2$ is given by (6). In the gauge $\bar D_i|\phi\rangle = 0$ and $L_{12}|\phi\rangle = 0$ we are left with

$$\Big(\mathcal{D}^2 + \lambda^2(d(p-1)+2p-p^2+s)\Big)|\phi\rangle = 0\;. \tag{51}$$

To compare these equations with the previously known results, we reproduce here the higher spin equations of [14] written in a covariant gauge for arbitrary mixed-symmetry massless fields $\phi^{(h_1,\ldots,h_\nu)}$ corresponding to the AdS_d representations $D(E_0,\mathbf{s})$ with the energy E_0 saturating the unitary bound and spin $\mathbf{s}=(h_1,\ldots,h_\nu)$ (here h_l are lengths of rows of corresponding traceless $o(d-1)$ Young tableau and $\nu\equiv\nu_{AdS}$ is a rank of $o(d-1)$)

$$\Big(\mathcal{D}^2-\lambda^2(h_k-k-1)(h_k-k-2+d)+\lambda^2\sum_{l=1}^{\nu}h_l\Big)\phi^{(h_1,\ldots,h_\nu)}=0\;. \tag{52}$$

Here h_k is the length of upper rectangular block and k is the number of the bottom row in this block (*i.e.*, k is a height of the block). The case of two-column Young tableaux corresponds to

$$k=p\;,\quad h_k=2\;,\qquad h_l=\begin{cases}2\,, & 1\leq l\leq p\\ 1\,, & p<l\leq s\end{cases}\;. \tag{53}$$

Plugging these values into (52) one indeed arrives at the equations (51).

One should note that the equations (52) are true in even dimensions for any h_l and in odd dimensions when $h_{(d-1)/2}=0$ [14], and should not be taken for granted for selfdual and antiselfdual representations which appear for non-zero $\pm h_{(d-1)/2}$. In our formulation $h_{(d-1)/2}$ can take non-zero values and, in fact, enters the theory only through $|h_{(d-1)/2}|$. We suggest our equations describe a sum of irreps with $-h_{(d-1)/2}$ and $+h_{(d-1)/2}$ and the duality condition (25) discussed above may help to extract selfdual and antiselfdual parts corresponding to different signs of $h_{(d-1)/2}$.

Discuss now the flat limit $\lambda=0$ of the field equations (50). It turns out that new symmetry $\delta|\phi\rangle=\partial_1|S\rangle$ appears in addition to that defined by (48) and the general gauge transformation takes now the form

$$\delta|\phi\rangle=\partial_1|S\rangle+\Big(\partial_2-\frac{1}{s-p+1}\,\partial_1\,T_{21}\Big)|\Lambda\rangle\;, \tag{54}$$

where Fock vector $|S\rangle$ is associated with the tensor $S^{(s-1,p)}$ (3) and the operators ∂_i are obtained from D_i-s (44) by replacing $\mathcal{D}^a\rightarrow\partial^a$. One can check directly that the equations (50) are do invariant under the gauge transformation (54) with the gauge parameter $|S\rangle$ at $\lambda=0$. However, the simplest way to prove the invariance is to observe that the auxiliary field being expressed through the physical field is an invariant expression with respect to $\delta|\phi\rangle=\partial_1|S\rangle$. Consequently, this additional invariance follows for the equations of motion on the flat background.

Note that starting from the equations of motion (50) one can develop an extended theory in the AdS_d space which realizes missing symmetry $|S\rangle$ as Stueckelberg symmetry [8]. To this end one should perform a shift $|\phi\rangle\rightarrow|\phi\rangle+D_1|\chi\rangle$ inside the equation of motion (50) with $|\chi\rangle$ having Young symmetry type of $|S\rangle$. Besides its own symmetries with derivatives, the Stueckelberg field $|\chi\rangle$ will be transformed as $\delta|\chi\rangle=\lambda|S\rangle$. The resulting equation involving $|\phi\rangle$ and $|\chi\rangle$ will be invariant under restored gauge symmetry with parameters $|\Lambda\rangle$ and $|S\rangle$. To obtain the equation for the Stueckelberg field $|\chi\rangle$, one should make the same shift inside the tensor $\lambda\,tr\,|T\rangle=$

0 (35). As a result one will get the second-order equation for the Stueckelberg field containing first-order terms with respect to the field $|\phi\rangle$. Then, the role of the condition (35) becomes quite clear. This is the differential condition (Noether identity) for a leftover symmetry with the parameter $|S\rangle$ (for more details see [8]). In the flat limit equations become diagonal with respect to fields $|\phi\rangle$ and $|\chi\rangle$ and describe a sum of independent higher spin fields.

5 Concluding remarks

In this paper we described higher spin fields in AdS_d corresponding to arbitrary two-column Young tableaux. Our approach is a particular realization of the general scheme proposed in [13] for generic mixed-symmetry fields. The method employs the set of p-forms with antisymmetric tangent indices having different dynamical interpretation. So, one distinguishes between the physical field and the auxiliary field and the latter is expressed in terms of the physical field by virtue of its equations of motion. The system has the gauge symmetry relevant to the anti-de Sitter background and reveals an additional symmetry in the flat limit $\lambda = 0$.

To conclude, it is worth to comment that the flat limit of the action functional (27) may give rise to the dual descriptions of higher spin fields in Minkowski space [15, 16, 6, 17]. The particular example is the three-cell "hook" field in AdS_5. In the flat limit the massless "hook" field decomposes into a couple of spin-2 massless fields because massless fields in $5d$ Minkowski space are exhausted by totally symmetric fields. As is easily seen from the equations (50) (see also Ref. [13]), these spin-2 massless fields are formulated in terms of dual variables [1, 15, 17]. In particular, the curvature tensors (33) and (34) become related by the duality transform as in [15].

Acknowledgements

I am grateful to O.V. Shaynkman and M.A. Vasiliev for useful discussions. Also, I would like to thank O.V. Shaynkman for careful reading of the manuscript and useful remarks. This work was partially supported by RFBR Grants No 02-02-16944, MAC No 03-02-06462, LSS-1578-2003-2 and the Landau Scholarship Foundation, Forschungszentrum Jülich.

References

[1] T. Curtright, Phys. Lett. B **165** (1985) 304.

[2] C.S. Aulakh, I.G. Koh, S. Ouvry, Phys. Lett. B **173** (1986) 284.

[3] J.M.F. Labastida and T.R. Morris, Phys. Lett. B **180** (1986) 101; J.M.F. Labastida, Phys. Rev. Lett. **58** (1987) 531; Nucl. Phys. B **322** (1989) 185.

[4] W. Siegel and B. Zwiebach, Nucl. Phys. B **282** (1987) 125.

[5] C. Burdik, A. Pashnev and M. Tsulaia, Mod. Phys. Lett. A **16** (2001) 731, hep-th/0101201; Nucl. Phys. Proc. Suppl. **102** (2001) 285, hep-th/0103143.

[6] X. Bekaert and N. Boulanger, "Tensor gauge fields in arbitrary representations of GL(D,R): duality and Poincare lemma", hep-th/0208058; Phys. Lett. B **561** (2003) 183, hep-th/0301243.

[7] P. de Medeiros and C. Hull, JHEP **0305** (2003) 019, hep-th/0303036.

[8] L. Brink, R.R. Metsaev and M.A. Vasiliev, Nucl. Phys. B **586** (2000) 183, hep-th/0005136.

[9] R.R. Metsaev, Phys. Lett. B **531**, (2002) 152, hep-th/0201226.

[10] T. Biswas and W. Siegel, JHEP **0207** (2002) 005, hep-th/0203115.

[11] Y.M. Zinoviev, "On massive mixed symmetry tensor fields in Minkowski space and (A)dS", hep-th/0211233.

[12] Y.M. Zinoviev, "First Order Formalism for Mixed Symmetry Tensor Fields", hep-th/0304067.

[13] K.B. Alkalaev, O.V. Shaynkman, M.A. Vasiliev, "On the Frame-Like Formulation of Mixed-Symmetry Massless Fields in $(A)dS_d$", hep-th/0311164.

[14] R.R. Metsaev, Phys. Lett. B **354** (1995) 78; Phys. Lett. B **419** (1998) 49, hep-th/9802097; Talk given at International Seminar on Supersymmetries and Quantum Symmetries (Dedicated to the Memory of Victor I. Ogievetsky), Dubna, Russia, 22-26 Jul 1997, hep-th/9810231.

[15] C. Hull, JHEP **0109** (2001) 027, hep-th/0107149.

[16] P. de Medeiros and C. Hull, Commun. Math. Phys. **235** (2003) 255, hep-th/0208155.

[17] N. Boulanger, S. Cnockaert and M. Henneaux, JHEP **0306** (2003) 060, hep-th/0306023.

[18] K. Stelle and P. West, Phys. Rev. D **21** (1980) 1466; C.R. Preitschopf and M.A. Vasiliev, The Superalgebraic Approach to Supergravity, in Proceedings of 31st International Ahrenshoop Symposium On The Theory Of Elementary Particles, Berlin, Wiley-VCH, 1998, 496p, hep-th/9805127.

[19] M.A. Vasiliev, "Higher Spin Gauge Theories: Star-Product and AdS Space"; Contributed article to Golfand's Memorial Volume"Many faces of the superworld", ed. by M.Shifman, World Scientific Publishing Co Pte Ltd, Singapore, 2000, hep-th/9910096; Int. J. Mod. Phys. D **5** (1996) 763, hep-th/9611024.

[20] P.K. Townsend, K. Pilch and P. van Nieuwenhuizen, Phys. Lett. B **136** (1984) 38 [Addendum-ibid. **137B** (1984) 443].

[21] V.E. Lopatin and M.A. Vasiliev, Mod. Phys. Lett. A **3** (1988) 257.

[22] S.W. MacDowell and F. Mansouri, Phys. Rev. Lett. **38** (1977) 739 [Erratum-ibid. **38** (1977) 1376]; F. Mansouri, Phys. Rev. D **16** (1977) 2456.